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Microscopic theory of dipole-exchange spin waves in magnetic multilayers

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Abstract. A microscopic model is employed to calculate the spectrum of dipole-exchange spin waves in multilayers in which thin ferromagnetic films are separated by non-magnetic spacers. Two configurations are considered: in one the films have magnetizations parallel to each other, in the other the magnetizations are antiparallel. The calculations extend a previous microscopic formalism that allows the calculation of the dipole-exchange spin wave spectrum in thin films. The results show the splitting of the frequency bands and the mode mixing caused by the dipolar interaction between the films as a function of spacer thickness.

PACS. 75.30.Ds Spin waves – 75.40.Gb Dynamic properties (dynamic susceptibility, spin waves, spin diffusion, dynamic scaling, etc.) – 75.70.-i Magnetic properties of thin films, surfaces, and interfaces

1 Introduction

Layered systems can display interesting properties that are not observed in single films or in bulk materials. Among these new nanostructured systems, layered films of magnetic materials have attracted a great deal of attention due to their possible use in the development of novel microelectronic devices and have thus been intensively investigated, both theoretically and experimentally [1]. The dynamic properties of magnetic multilayers, in particular, can be quite distinct from those of single films. For a single ferromagnetic film with magnetization parallel to the surface, calculations of magnetostatic modes show that, apart from an infinite set of solutions that describe bulk spin wave (SW) modes, solutions corresponding to surface waves can also be found. These surface waves, also known as Damon-Eshbach (DE) modes, have frequencies above the bulk modes and propagate only in a restricted set of directions [2]. Calculations of the magnetostatic modes of structures in which magnetic layers are interspersed with vacuum or with a non-magnetic medium have shown that their spin wave spectrum can be significantly distinct from the spectrum of a single film [3–6]. In such multilayers, the long-wavelength magnetostatic frequency branches are splitted into bands and shifted in relation to the single film results. The broadening of the modes was shown to depend on the spacer thickness and on the number of magnetic layers in the structure. These effects were proven to be caused by a coupling of the SW modes of each film due to the long-range dipolar interactions across the non-magnetic spacer.

The magnetostatic formalism provides a good description of the SW dynamics at long wavelengths. However, there is experimental evidence that exchange effects can influence the dispersion of SWs even for relatively long wavelengths [7–10]. The exchange interaction is also known to be responsible for a modification of the DE modes and for a quantization of the volume modes in thin films (i.e. with thicknesses of the order of the exchange SW wavelengths) in comparison with thick films. In the last decade, several papers have investigated the wavevector regime in which both dipolar and exchange effects can significantly influence the spin dynamics in thin films using a microscopic approach, for both ferromagnets and antiferromagnets [11–16]. So far, however, the dipolar coupling between separate films has not been included in the context of these microscopic calculations.

In this paper we perform an extension of the previous models by developing a microscopic theory of dipole-exchange spin waves propagating in magnetic layered systems. Specifically, we consider a magnetic medium that consists of a set of ferromagnetic thin films separated by non-magnetic spacers. A formalism is developed, which allows us to obtain dispersion relations of the localized surface SW as well as the quantized volume SW modes for all wave vectors, whereas the existing macroscopic approaches are valid only in the regime of small wavevectors and for thicker films. The model is based on a Hamiltonian representation of the dipolar terms, with the long-range summations being evaluated by means of the technique described in reference [17]. The paper is structured as follows: in Section 2 the structure of the multilayer is described and a microscopic Hamiltonian for the system is introduced. Expressions for the equations of motion for

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the operators are presented in Sections 3 and 4 for parallel and antiparallel spin configurations, respectively. In Section 5, numerical results are presented for multilayers of $GdCl_3$ and EuO. Section 6 has a summary of the results and the conclusions.

2 Hamiltonian and multilayer geometry

Let us consider a system containing N_l ferromagnetic films separated by non-magnetic spacers, as pictured in Figure 1. In the following discussion, the ferromagnets and spacers are referred to as *films*, whereas the spin layers in each ferromagnet are referred to as *atomic layers*. The films have a simple cubic crystal structure, with lattice constant a and have ideal interfaces corresponding to (001) crystal planes. Each of the magnetic films is a single domain and contains N_m atomic layers (in the xy plane) with localized spins, which are assumed to be aligned in-plane and to interact by isotropic exchange with its nearest-neighbors and via dipolar coupling with all other spins in the structure. All magnetic films have the same thickness and composition and every site in the magnetic lattice has the same number of nearest neighbors, with the exception of the uppermost and bottom films, which are assumed to have vacuum interfaces. The relative orientation of the magnetizations of neighboring films are taken to be either parallel or to alternate along the y direction. The spacer films are all non-magnetic and have the same thickness d, which is expressed in multiples of the lattice parameter a. The Hamiltonian for the system is

$$\mathcal{H} = -\sum_{i,j} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j - g\mu_B \sum_i H_0 S_i^z + (g\mu_B)^2 \sum_{\alpha,\beta} \sum_{l,m} D_{lm}^{\alpha\beta} S_l^\alpha S_m^\beta, \quad (1)$$

where J_{ij} is the exchange between nearest-neighbor sites i and j in the same film H_0 is the Zeeman field, taken as parallel to the $z-$direction. The last term in the Hamiltonian represents the contribution of the dipolar interaction. In phenomenological theories [18], this contribution is usually expressed in terms of a demagnetizing field $\mathbf{H}_d$. In contrast, in the present model the dipolar interaction is obtained in terms of the microscopic coupling between localized moments, with the $D_{lm}^{\alpha\beta}$ being the long-range dipolar coupling coefficient between any sites l and m in the same or different films. The α and β indices denote components x, y or z; g is the Landé factor and μ_B is Bohr's magneton. The expression for the dipolar factors is

$$D_{lm}^{\alpha\beta} = \frac{\left[|\mathbf{r}_{lm}|^2 \delta_{\alpha\beta} - 3r_{lm}^\alpha r_{lm}^\beta\right]}{|\mathbf{r}_{lm}|^5}, \quad (2)$$

here the vector $\mathbf{r}_{lm} = \mathbf{r}_l - \mathbf{r}_m$ connects magnetic sites in the lattice. In order to calculate the SW frequencies, one can write down the equations of motion for the operators in the films. These are then transformed to a representation involving a two-dimensional in-plane wave vector

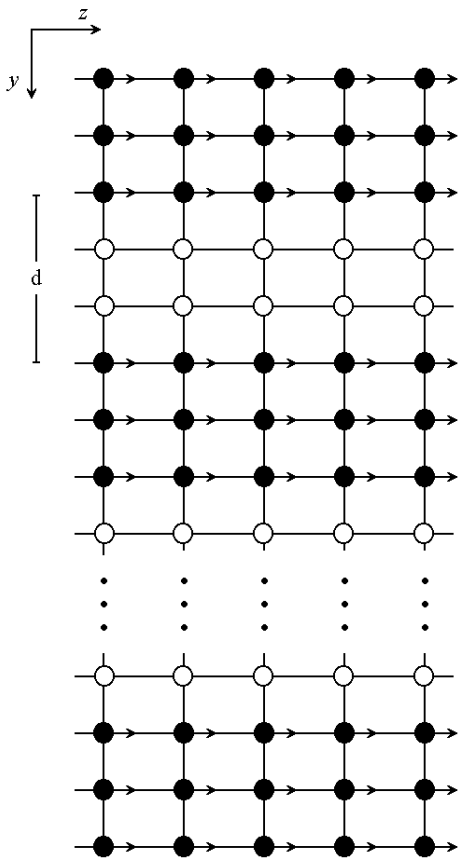

Fig. 1. Schematic depiction of a layered magnetic structure. In this figure, the ferromagnetic thin films have spins parallel to each other and are separated by non-magnetic spacers of thickness d.

$\mathbf{k} = (k_x, k_z)$ parallel to the film surfaces. The resulting system of equations can be solved numerically for the frequency ω of the modes. The Fourier amplitudes of the dipolar terms can be expressed in terms of rapidly converging summations as shown in reference [15]. For multilayers, these expressions for the amplitudes are modified by the presence of the spacers and by changing the relative orientation of magnetization of the films. However, by increasing the thickness of the spacers, the ferromagnets eventually become uncoupled, and the resulting SW spectrum must then reproduce the results for the single film.

3 Parallel configuration

In this case, the system has the magnetizations of the films parallel to each other. To each atomic layer containing magnetic sites is assigned an index n, such that $n = 1, 2, \cdots, N_m \times N_l$, where $N_m \times N_l$ is the total number of magnetic layers in the structure. The spin Hamiltonian can be rewritten in terms of boson creation and annihilation operators by means of the Holstein-Primakoff transformation [19] which, for low temperatures (i.e. $T \ll T_c$,

where T_c is the Curie temperature of the ferromagnets) can be written as:

$$S_i{}^+ \approx \sqrt{2S}\, a_i, \tag{3}$$

$$S_i{}^- \approx \sqrt{2S}\, a_i{}^\dagger, \tag{4}$$

$$S_i{}^z = S - a_i{}^\dagger a_i, \tag{5}$$

where $a_i{}^\dagger$ and a_i are boson creation and annihilation operators, respectively. The transformed Hamiltonian can then be written as

$$\mathcal{H} = -\sum_{i,j} S J_{ij}\left(a_i^\dagger a_j - a_i^\dagger a_i\right) + g\mu_B\, H_0 \sum_l a_l^\dagger a_l + (g\mu_B)^2 S \frac{1}{4}\sum_{l,m}\Big[A_{lm} a_l a_m + \overline{A}_{l,m} a_l^\dagger a_m^\dagger - 2D_{l,m}^{zz}\left(a_l^\dagger a_m + 2a_l^\dagger a_m\right)\Big]. \tag{6}$$

We have retained only the terms in the Hamiltonian that have products of two operators, which describe non-interacting SW and, for any two generic sites p and p',

$$A_{pp'} \equiv D_{p,p'}^{xx} - D_{p,p'}^{yy} - 2iD_{p,p'}^{zz}, \tag{7}$$

$$\overline{A}_{pp'} \equiv D_{p,p'}^{xx} - D_{p,p'}^{yy} + 2iD_{p,p'}^{zz}. \tag{8}$$

Next, the equations of motion for the creation and annihilation operators are obtained, and are afterwards transformed to a representation involving a two-dimensional (2D) in-plane wave vector $\mathbf{k}_n = (k_x, k_y)$, where n is an index assigned to each *magnetic* atomic layer of the system. The Fourier transforms of the dipole sums in the Hamiltonian are similar to the terms calculated for a single ferromagnetic film [15,17], with additional terms including extra distance factors, which correspond to the dipolar interaction between spins in different films. Specifically, the dipole sums that describe the interactions between spins in different layers contain an exponential term $e^{-2|y|\gamma_{lm}}$, where $\gamma_{lm} = \sqrt{(\pi l/a + q_x/2)^2 + (\pi m/a + q_z/2)^2}$ and $|y|$ is the distance between the layers. Thus, for layers in different films, the extra distance corresponding to the thickness of the spacers between the films must be added to $|y|$. The equations of motion become

$$\sum_{n'}\left\{M_{n,n'} a_{n'}^\dagger(\mathbf{k}) + N_{n,n'}\, a_{n'}(-\mathbf{k})\right\} = 0,$$

$$\sum_{n'}\left\{N_{n',n} a_{n'}^\dagger(\mathbf{k}) + M_{n,n'}\, a_{n'}(-\mathbf{k})\right\} = 0, \tag{9}$$

with

$$M_{n,n'} = \Big\{g\mu_B H_0 + S\Big[u_n(0) - u_n(\mathbf{k}) + v_{n,n-1} + v_{n,n+1} + (g\mu_B)^2 \sum_{n''} D_{n,n''}^{zz}(0)\Big]\Big\}\delta_{n,n'} - S\left[v_{n,n+1}\delta_{n',n+1} + v_{n,n-1}\delta_{n',n-1}\right] - S\,(g\mu_B)^2\, D_{n,n'}^{zz}(\mathbf{k}), \tag{10}$$

and

$$N_{n,n'} = \frac{1}{4} S\,(g\mu_B)^2\,\left[D_{n,n'}^{xx}(\mathbf{k}) - D_{n,n'}^{yy}(\mathbf{k}) - 2iD_{n,n'}^{xy}(\mathbf{k})\right]. \tag{11}$$

The factor $u_n(\mathbf{k}) = 2\,J[\cos(k_x\,a) + \cos(k_z\,a)]$ is the Fourier amplitude of the exchange interaction between sites in the same atomic layer and $v_{n,n}(\mathbf{k}) = SJ$ is the exchange amplitude for interactions between sites in adjacent atomic layers. In this paper we assume that J has the same value for all pairs of nearest-neighbor spins. However, one can easily modify the expressions to include different strengths for the exchange constant at the ferromagnet-spacer or ferromagnet-vacuum interfaces. The SW frequencies are then calculated by solving the eigenvalue equation

$$\chi(\mathbf{k})\,\mathbf{A}(\mathbf{k}) = 0, \tag{12}$$

where

$$\chi(\mathbf{k}) = \begin{pmatrix} \mathbf{M}(\mathbf{k}) & 2\mathbf{N}(\mathbf{k}) \\ 2\mathbf{N}(-\mathbf{k}) & \mathbf{M}(-\mathbf{k}) \end{pmatrix}. \tag{13}$$

This matrix has $4N \times 4N$ elements, where $N \equiv N_m \times N_l$ is the total number of magnetic atomic layers. The column matrices are defined as

$$\mathbf{A}_\mathbf{k} = \begin{pmatrix} \mathbf{a}_\mathbf{k}^\dagger \\ \mathbf{a}_\mathbf{k} \end{pmatrix}, \tag{14}$$

with

$$\mathbf{a}_\mathbf{k}^\dagger = \begin{pmatrix} a_{\mathbf{k},1}^\dagger \\ a_{\mathbf{k},2}^\dagger \\ \vdots \\ a_{\mathbf{k},N}^\dagger \end{pmatrix} \quad , \quad \mathbf{a}_\mathbf{k} = \begin{pmatrix} a_{\mathbf{k},1} \\ a_{\mathbf{k},2} \\ \vdots \\ a_{\mathbf{k},N} \end{pmatrix}. \tag{15}$$

The matrices $\mathbf{M}$ and $\mathbf{N}$ can be expressed as

$$\mathbf{M}(\mathbf{k}) = \begin{pmatrix} \mathbf{M}^{11}(\mathbf{k}) & \mathbf{M}^{12}(\mathbf{k}) & \cdots & \mathbf{M}^{1N_l}(\mathbf{k}) \\ \mathbf{M}^{21}(\mathbf{k}) & \mathbf{M}^{22}(\mathbf{k}) & \cdots & \mathbf{M}^{2N_l}(\mathbf{k}) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}^{N_l 1}(\mathbf{k}) & \mathbf{M}^{N_l 2}(\mathbf{k}) & \cdots & \mathbf{M}^{N_l N_l}(\mathbf{k}) \end{pmatrix} \tag{16}$$

and

$$\mathbf{N}(\mathbf{k}) = \begin{pmatrix} \mathbf{N}^{11}(\mathbf{k}) & \mathbf{N}^{12}(\mathbf{k}) & \cdots & \mathbf{N}^{1N_l}(\mathbf{k}) \\ \mathbf{N}^{21}(\mathbf{k}) & \mathbf{N}^{22}(\mathbf{k}) & \cdots & \mathbf{N}^{2N_l}(\mathbf{k}) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{N}^{N_l 1}(\mathbf{k}) & \mathbf{N}^{N_l 2}(\mathbf{k}) & \cdots & \mathbf{N}^{N_l N_l}(\mathbf{k}) \end{pmatrix}. \tag{17}$$

The upper indices in equations (16, 17) identify the films, which means that the $N_m \times N_m$ matrices $\mathbf{M}^{tt}(\mathbf{k})$ and $\mathbf{N}^{tt}(\mathbf{k})$ (where t ranges from 1 to N_l), have the information regarding the interactions between spins within the same

film, whereas the remaining matrices describe couplings between spins in different films, and thus do not contain exchange terms. As mentioned above, the dipolar terms for a matrix $\mathbf{M}^{st}(\mathbf{k})$ (where $s \neq t$, and $1 \leq s \leq N_l$) have additional distance factors $|s-t|d$, due to the presence of the non-magnetic spacers of width d. These distance factors also are included in the summation of $D^{zz}_{n,n''}(0)$ in equation (10).

4 Antiparallel configuration

In this configuration, the direction of magnetization alternates along the y direction. Such configuration can be obtained by using magnetic films of materials with different coercive fields, or by taking advantage of a long-range antiferromagnetic coupling between the films. In this paper we consider an ideal case in which all magnetic layers have the same composition and the coupling between different films is purely of dipolar nature. Such ideal antiparallel structures have been previously investigated in the framework of a continuum model [5]. The microscopic approach for calculating the dipole-exchange SW spectra is similar to the procedure for the parallel case. However, care must be taken to correctly introduce the creation and annihilation operators, since the system now can be described as being divided into two sublattices: one having spins in each atomic layer aligned along the "up" direction (i.e. the direction of positive z), and the other having spins along the "down" direction. For films magnetized in the "up" direction, one can utilize the expressions of equations (3–5). For films with spins in the "down" direction, one can define

$$S_j{}^{+} \approx \sqrt{2S}\, a_j{}^{\dagger}, \tag{18}$$

$$S_j{}^{-} \approx \sqrt{2S}\, a_j, \tag{19}$$

$$S_j{}^{z} = -S + a_j{}^{\dagger} a_j. \tag{20}$$

The equations of motion are similar to those obtained for the parallel case, with the equations (10, 11) redefined as

$$\begin{aligned} M^{st}_{n,n'} = & \Big\{ (-1)^{s+1} g\mu_B H_0 + S \Big[u_n(0) - u_n(\mathbf{k}) \\ & + v_{n,n-1} + v_{n,n+1} + (g\mu_B)^2 \sum_{n''} D^{zz}_{n,n''}(0) \Big] \Big\} \delta_{n,n'} \\ & - S \left[v_{n,n+1}\delta_{n',n+1} + v_{n,n-1}\delta_{n',n-1} \right] \\ & - (-1)^{s-t} S\, (g\mu_B)^2\, D^{zz}_{n,n'}(\mathbf{k}), \end{aligned} \tag{21}$$

and

$$\begin{aligned} N^{st}_{n,n'} = (-1)^{s-t} \frac{1}{4} S\, (g\mu_B)^2 \Big[& D^{xx}_{n,n'}(\mathbf{k}) \\ & - D^{yy}_{n,n'}(\mathbf{k}) - 2iD^{xy}_{n,n'}(\mathbf{k}) \Big], \end{aligned} \tag{22}$$

where the $(-1)^{s-t}$ and $(-1)^{s+1}$ factors arise due to the opposite orientations of the spins in films with indices odd an even.

5 Numerical results

Numerical results were obtained for structures with ferromagnetic films of $GdCl_3$ ($T_c \approx 2.2$ K) and EuO ($T_c \approx 69$ K). Ultrathin films of these materials have been recently studied in the framework of a microscopic model, that yielded results for both the linear SW spectrum as well as nonlinear effects [15]. The $GdCl_3$, in particular, has a large ratio of dipolar to exchange strengths, which means that in such ferromagnet the effects due to both dipolar and exchange couplings may extend to a larger wavevector range than in other systems. For the EuO we used effective exchange parameters, since this material does not normally have a simple cubic structure. Also for the EuO, for the sake of simplicity we assumed only nearest neighbor exchange coupling. The parameters for the dipolar coupling strength (given in terms of the bulk saturation magnetization) and the exchange field were, for the $GdCl_3$, $4\pi M = 0.82$ T and $H_{ex} = 0.54$ T, respectively, and $4\pi M = 2.4$ T and $H_{ex} = 38$ T for the EuO, where in both cases $g\mu_B H_{ex} = 6SJ$.

Figure 2 shows a dispersion relation of the SW modes of a $GdCl_3$ structure with 3 ferromagnetic thin films in the parallel configuration, each one comprising 25 atomic layers and separated by two non-magnetic spacers with $d = 35a$, and under an external field $H_0 = 0.36$ T. The frequencies of the lowest SW modes are plotted against $k_x a/\pi$, for the Voigt geometry ($k_y = 0$) and given in units of GHz (using $\gamma = 28$ GHz/T). One can clearly see the splitting of the DE modes, caused by the long range coupling of the films at small wavevectors. This behavior is in good agreement with the results obtained by previous magnetostatic theories (see, e.g., Ref. [4]). These earlier approaches, however, by treating the films as continuum media, were not able to display the significant mode mixing effects between the surface modes and the discrete bulk branches which are evident in Figure 2. The graph also shows the effect of the dipolar coupling on the bulk branches. For large wavevectors, the volume modes correspond to the quantized frequencies that are obtained from exchange-dominated models for single films. On the other hand, for small k_x, each of these quantized modes are split into three separate branches. In fact, in the dipole-exchange regime the excitations at small $\mathbf{k}$ can be characterized as collective modes of the entire structure. In agreement with the results for the dipole-exchange SW spectrum of a single film, the lowest magnetostatic frequencies for the multilayers are found to approach 18.25 GHz for $k_x = 0$, which is close to the result of $[H_0(H_0+4\pi M)]^{1/2}$ obtained from the continuum theories.

A dispersion relation for a structure with a larger number of films is shown in Figure 3. This multilayer is in the parallel configuration and contains 10 films of $GdCl_3$ with 15 atomic layers, each film separated from its neighbors by a distance $d = 30a$, for $H_0 = 0.36$ T. The graph shows discrete branches at large wavevectors being broadened into frequency bands at smaller wavevectors. The surface band is found in the gaps between the volume bands. The localized nature of these modes is deduced from the fact that, as the wavevector increases, they tend to merge into the

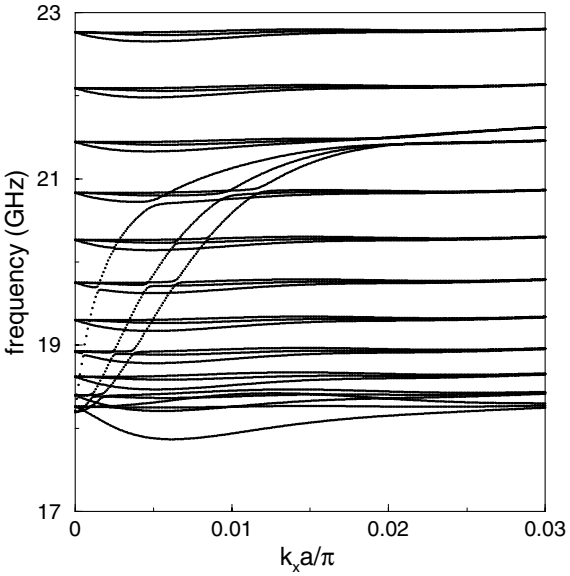

Fig. 2. Spin wave dispersion relation for a $GdCl_3$ structure with three films, in the parallel configuration, for an external field $H_0 = 0.36$ T. In this case each ferromagnet has 25 atomic layers, whereas the spacers have thicknesses $d = 35a$, where a is the lattice parameter of the ferromagnets.

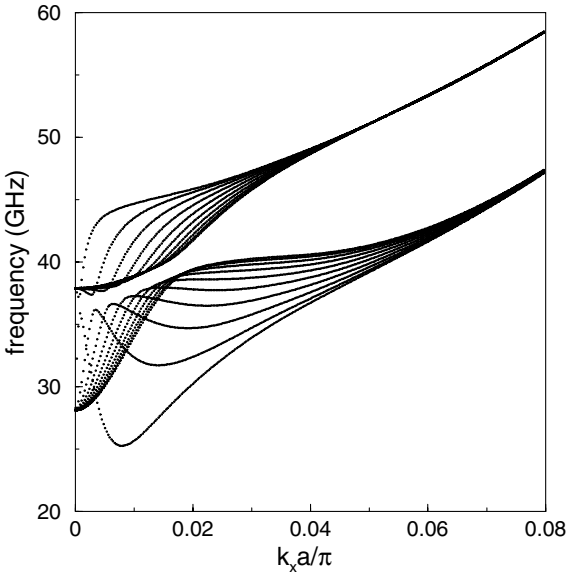

Fig. 4. Spin wave dispersion relation for a EuO structure with 10 films, in the parallel configuration, for an external field $H_0 = 0.36$ T. In this case each ferromagnet has 16 atomic layers, whereas the spacers have thicknesses $d = 8a$, where a is the lattice parameter of the ferromagnets.

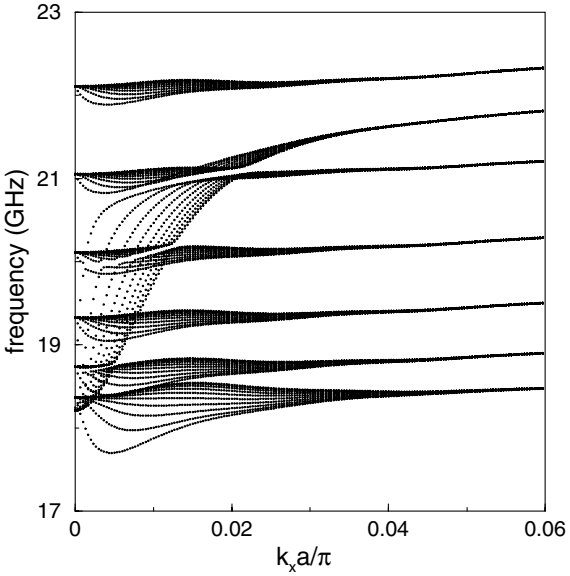

Fig. 3. Spin wave spectrum for a $GdCl_3$ structure with 10 films in the parallel configuration, where the magnetic films have each 15 atomic layers, and the spacers have thicknesses $d = 30a$.

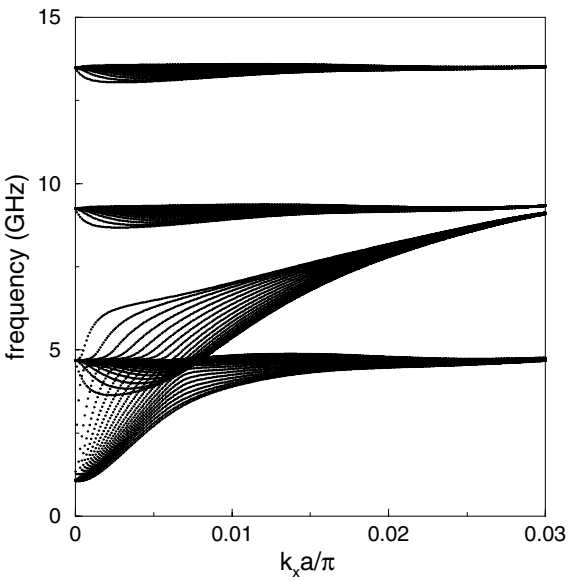

Fig. 5. Spin wave spectrum at zero external field for a $GdCl_3$ structure with 20 films, in the antiparallel configuration, where the magnetic layers have 5 atomic layers each, and the spacers have thicknesses $d = 50a$.

DE modes of a single film. One can notice the presence of small gaps in the SW spectrum, for SW frequencies around 18.7 GHz, 20.0 GHz and 20.8 GHz. These gaps arise due to a mode repulsion effect between the volume and surface modes, a behavior that can also be observed in Figure 2.

The mode repulsion effect is more evident in the dispersion graph shown in Figure 4. This result was obtained for a 10 films EuO structure in the parallel configuration, in which the atomic layers in the ferromagnets had 16 atomic layers each, whereas the spacers had thickness $d = 8a$. As in the previous results, an external field of 0.36 T was assumed. For $\mathbf{k} = 0$, the lowest SW frequency approaches 28 GHz, in agreement with the value predicted by the continuum theories. However, the graph also shows a minimum in the lowest dispersion branch of ≈ 25 GHz at small wavevectors.

Figure 5 shows the SW spectrum for a $GdCl_3$ multilayer in the antiparallel configuration. The structure has

20 films, each with 5 atomic layers. The spacers have $d = 50a$ each. The external field is zero in this case, since the existence of an applied field can destroy the magnetization configuration. As in the previous results, the discrete frequency branches are broadened into bands at small wavevectors due to the dipolar coupling between the films. In this case the thickness of the spacers was set to a larger value than in the parallel configuration calculations. In fact, for both parallel and antiparallel configurations, the results for the $GdCl_3$ indicate that if the thickness d of the spacers is comparable to the thickness of the magnetic films, an instability of the microscopic spin configuration may arise. This instability is signaled by the presence of non-physical solutions corresponding to $\omega = 0$ for non-zero wavevectors and occurs as a consequence of the internal bias field produced by the neighboring films. This field can strongly affect the spins in the ferromagnet-spacer interfaces, since these are exchange-coupled to a smaller number of neighbors in comparison with the spin sites in the interior of the ferromagnets. The internal field can thus cause a reorientation of the spins out of the collinear state at the interfaces. The instability of the collinear configuration was found to depend on the number of films and on the thickness of the spacers. It can be removed by increasing the thickness d of the spacers, by applying an external field (for the parallel configuration) or through the addition of surface anisotropies or a modified surface exchange constant to the model, which can effectively pin the interface spins.

The effect of the external field is shown in the graph of Figure 6, which displays the SW frequencies as a function of H_0. This result was obtained for a structure with 10 films, each having 7 atomic layers, with $d = 7a$, in the parallel configuration, for $k_x a = 0.005\pi$. As the external field approaches zero, the lowest SW branch goes to zero, which indicates a possible instability of the collinear state. As the field increases, the collinear state is recovered and the SW frequencies begin to show a linear dependence on the field.

Two graphs in Figure 7 show results for the lowest SW frequency branches as a function of spacer thickness, for multilayers of EuO at zero external field and for $k_x a/\pi = 0.01$. The upper plot (a) was obtained for the parallel configuration, whereas the lower plot (b) was calculated for the antiparallel configuration. In both cases the structures have 10 magnetic films and the ferromagnets have 10 atomic layers each. The effects of the long-range coupling are found to be significant, in this wavevector range, even for spacer widths that are much larger than the individual film thicknesses. For thin spacers, the SW branches are broadened into bands, which is particularly wide in the antiparallel case. As the thickness increases, the branches merge into the quantized modes predicted by exchange-dominated models at $d \approx 200a$. For larger values of $\mathbf{k}$, the width of the bands is also found to decrease, as a consequence of the weaker coupling of the modes. The instability of the microscopic spin configuration does not occur in this case, due to the smaller strength of the dipolar coupling in comparison with the exchange interaction

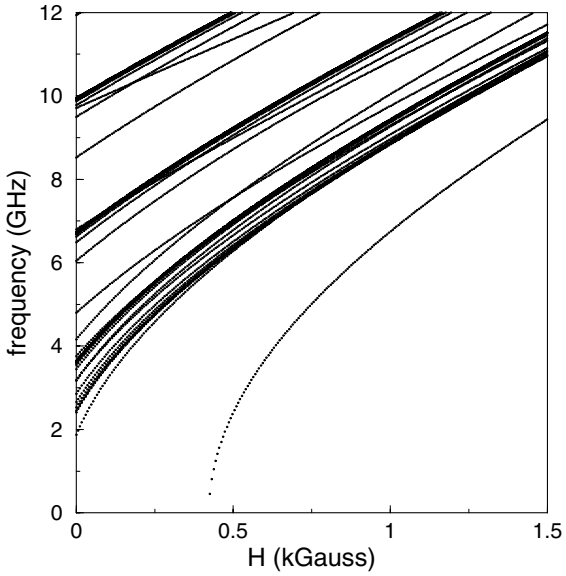

Fig. 6. Lowest SW frequencies as a function of external field for a $GdCl_3$ multilayer in the parallel configuration, for $k_x a = 0.005\pi$. The structure has 10 magnetic films, each with 7 atomic layers. In this case, $d = 7a$.

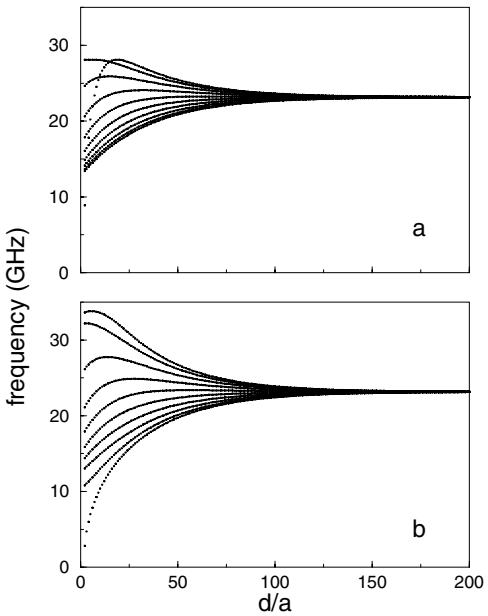

Fig. 7. Lowest SW frequencies as a function of spacer thickness for a EuO multilayers, for $k_x a = 0.01\pi$ in the parallel (a) and antiparallel (b) configurations. The structure has 10 films and each film has 10 atomic layers. The external field was set to zero.

in the EuO. The data in both graphs agree well with the results of the continuum theories (see, e.g. [3]).

The presence of gaps in the SW spectrum can be seen in Figure 8, which shows the lower frequency branches as a function of the total number of films, for multilayers of $GdCl_3$. In this case the structures are in the parallel configuration and under an external field of 0.36 T. The magnetic films have thickness $5a$, the distance between the films is $d = 5a$, and $k_x a = 0.01\pi$. The graph shows both

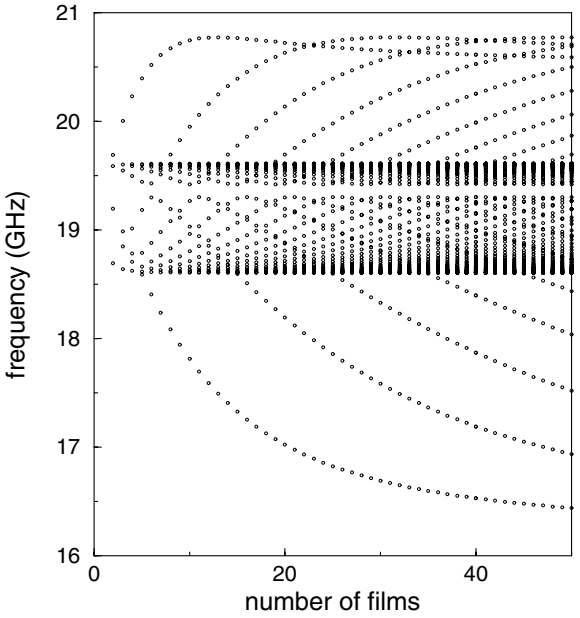

Fig. 8. Lowest SW frequencies as a function of the number of films, for a $GdCl_3$ multilayer in the parallel configuration. In this case the magnetic films have 5 atomic layers and the external field was set to 0.36 T.

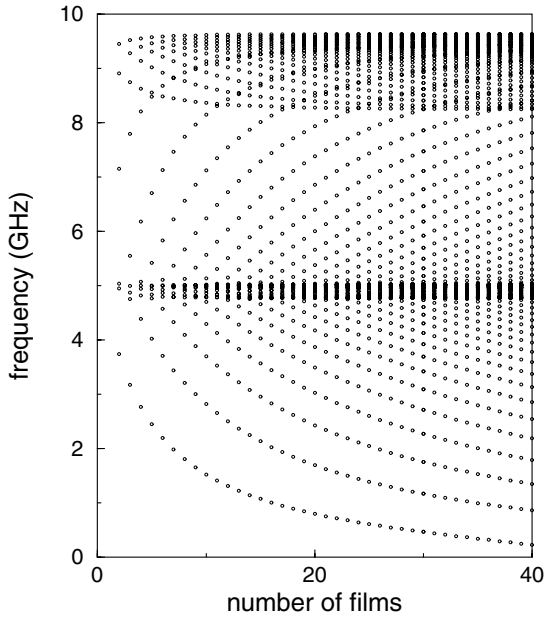

Fig. 9. Lowest SW frequencies as a function of the number of films, for a $GdCl_3$ multilayer in the antiparallel configuration. The magnetic films have 5 atomic layers and the external field was set to zero.

volume and interface modes, with the latter arising from the volume band as the number of films is increased. A small frequency gap is found between 19.3 and 19.4 GHz. Figure 9 shows the SW frequencies as a function of the total number of films for the antiparallel configuration of $GdCl_3$ multilayers. In this case, the external field was set to zero. The magnetic films have thickness $5a$, the distance between the films is $d = 20a$, and $k_x a = 0.01\pi$. Due to the larger distances between the films, the frequency bands are found to be narrower. The lower frequencies approach zero as the number of films increases, as a consequence of the internal field of the structure.

6 Conclusions

We have presented a microscopic model of magnetic multilayers based on a Hamiltonian formalism previously applied to the study of dipole-exchange SW in ultrathin ferromagnetic films. The present theory includes nearest-neighbor exchange coupling between localized spins in each magnetic layer of the structure, and also takes into account the long-range dipolar coupling between all spins in the medium. Numerical results for the dipole-exchange SW spectrum were presented for multilayers with parameters corresponding to the ferromagnets $GdCl_3$ and EuO. The data show the effect of the long-range nature of the dipolar interaction, which couples the SW modes in each film and thus causes significant modifications of the SW frequency branches, in comparison with the results for a single film. The results were obtained for two cases: a configuration with all films magnetized parallel to each other, and an antiparallel configuration, in which the magnetization alternates along the y direction. The differences in the dipolar field for each configuration were shown to give rise to distinct behaviors of the SW modes, such as the broadening of the SW branches into bands, which was found to be more pronounced in the antiparallel case. This behavior agrees with previous results obtained in the framework of theories based on a continuum model of ferromagnetic films. These theories, however, by assuming a uniform precession of the magnetization in each film, cannot take into account the influence of the volume SW modes. Thus, the microscopic nature of the present model causes the presence of features in the SW spectrum that cannot be accounted for by a continuum description, such as hybridization effects between the interface and the quantized volume frequency branches, which creates gaps in the SW spectrum. Another microscopic consequence of the dipole-dipole coupling is a possible reorientation transition of the spins at the ferromagnet-spacer interfaces, due to the internal bias field of the structure, that has a dependence on the thicknesses of the spacers and on the surface parameters. The reorientation of spins at the surfaces of magnetic films and multilayers is a phenomenon that has attracted considerable attention [20]. In phenomenological approaches, this behavior is obtained as a consequence of an external applied field or through the addition of surface induced exchange and anisotropy energy contributions. On the other hand, in the present calculation the reorientation results from the dynamic aspect of the dipolar interaction between the films. Theoretical extensions of this model could take into account the influence of interface roughness on the SW spectrum, structures with spacers with different thicknesses, other ferromagnets with different compositions and exchange interaction schemes, and the influence of a long-range exchange coupling, which has been observed in metallic multilayers.

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